Complex Analysis Study Guide Test 2(Draft!!)

The test is Friday March 17th. No calculators. Closed notes.

Material for test 1:

- Everything from the 1st test to this week.
- Multivalued functions
    - Identify when and why a function is multivalued
    - Be able to work with the two typical types of multivalued functions (roots and logs)
        - Determine branch points / cuts
        - Find values
        - Use them in equations
        - Understand and illustrate conceptually what's going on
- Complex integration
    - Integration on contours (closed or open, but typically open) by parametrizing the contour.
        - Be able to find parametric equations to describe any function
        - Be able to find parametric equations to describe common non-functions
        - Be able to use such parametric equations to find integrals (rectangular or polar, but usually rectangular)
    - Be able to find complex integrals (closed or open contours) using antiderivatives
        - Know when they apply (There are several theorems)
    - Be able to find complex integrals on closed contours using Cauchy's theorem.
    - Be able to find complex integrals on closed contours using Cauchy's Integral Formula

Practice problems

Below are a selection of problems from our textbooks that looks like reasonable problems that could appear on a test. An "easy" problem means that you should be able to jump right in and start solving it immediately. A "medium" problem means it is expected that you'll need to think a little before solving the problem. A "hard" problem means you'll need to think a lot and maybe work out some details before solving the problem.

Easy Problems

1. Let $w = z^{\frac{1}{3}}$. Suppose a person starts at $z = 1$ with $w(1) = 1$ and walks around the unit circle for a total of 720^o. What is $w(1)$ this time? (2.55)
2. Find the branch points of $f(z) = \left(\frac{z}{1-z}\right)^{\frac{1}{2}}$ (2.57)
3. Find the branch points of $(z^2 + 1)^{\frac{1}{3}}$ (2.85)
4. Find $\int_C (2y + x^2)dx + (3x - y)dy$ where C is the line connecting $3i$ and $2 + 4i$. (4.1c)
5. Find $\int_C \bar{z}dz$ where C is the line connecting $2i$ and $4 + 2i$. (4.2b)
6. Find $\int_C 2(z^2 + 3z)dz$ where C is the circle centered at $2 + 3i$ with radius 4. (4.12b)
7. Find $\int_C \frac{dz}{z-5}$ where C is the unit circle. (4.21a)
8. Find $\int_C \frac{dz}{z-5}$ where C is the triangle with corners $2 - i, 7 - 3i$, and $5 + 6i$ (4.21b)
9. Find all values of $i^{\frac{1}{2}}$ (R2.2.2a)
10. Find all values of $|i|^{\frac{1}{2}}$
11. Find $\int_C (1 + 2z + z^2)dz$ where C is the unit circle. (R2.4.2a)
12. Find $\int_C \left(\frac{1}{\left(z-\frac{1}{2}\right)^2}\right)dz$ where C is the unit circle. (R2.4.2b)
13. Find $\int_C \left(\frac{1}{z(z-2)}\right)dz$ where C is the unit circle. (R2.5.2a)
14. Find an open region on which $\sum_{k=1}^{\infty} z^k$ converges. (R3.1.5.a)

Medium problems

1. Find suitable branch cuts of $f(z) = \left(\frac{z}{1-z}\right)^{\frac{1}{2}}$ (2.57)
2. Find all the values of $\sin^{-1}(2)$. (2.79)
3. Find all the values of $\sinh^{-1}(2)$. (2.80)
4. Find $\int_C dz$ where C is the circle centered at $2 + 3i$ with radius 4. (4.12a)
5. Find $\int_C \frac{dz}{(z-5)^n}$ where C is the unit circle. (4.22a)
6. Find $\int_C \frac{dz}{(z-5)^n}$ where C is the triangle with corners $2 - i, 7 - 3i$, and $5 + 6i$ (4.22ish)
7. Find all branch points for the multidefined function $f(z) = \log\big((z-1)(z-2)\big)$. (R2.3.2.a)
8. Find $\int_C \frac{e^{iz}}{z(z-\pi)}dz$ where C is the annulus with inner radii 1 and outer radii 3. (R2.5.3a)
9. Find $\int_C \left(\frac{1}{(2z-1)^2}\right)dz$ where C is the unit circle. (R2.6.1b)
10. Let $f_n(z) = \frac{1}{z-n}$. Find $\lim_{n\to\infty} \int_C f_n(z)dz$ where C is a circle centered at the origin with radius 5. (R3.1.4a)
11. Let $f_n(z) = \frac{1}{z-\frac{1}{n}}$. Find $\lim_{n\to\infty} \int_C f_n(z)dz$ where C is a circle centered at the origin with radius 5. (R3.1.4bish)

Hard problems

1. The function $w(z) = e^{iz}$ is period. What is its period? Justify your answer. (2.60)
2. Find all values of 2^{1+i}.
3. Find a branch cut for the multidefined function $f(z) = \log\big((z-1)(z-2)\big)$. (R2.3.2.a)
4. Find $\int_{-1}^{1} z^{\frac{1}{2}}dz$ using the principal branch. (R2.4.4b)